

**A-Level Further Maths with Mathematics A level Guide**

**How Further Maths and Mathematics will be taught:**

A level Further Maths is integrated alongside Maths and taught through a series of interleaving topics with an initial focus on the core (pure) content of both A levels. Pure maths builds on aspects that have already been met at GCSE and introduces some new areas of study (more familiar to anyone who has studied level 2 Further Maths). Further Core content is by its nature more abstract and a stepping-stone towards the mathematics met in a Maths or Physics degree. The applied Statistics and Mechanics content in Further Maths builds on the Mathematics A level applied content, so our teaching plans are integrated. Further decision is taught to all Further Maths students in year 12 to give experience of all these three optional applied units, before students select their preferred two applied units in year 13.

Your teachers introduce each topic with an overview of the elements within it and its links with your prior learning and then take you through the techniques and reasoning behind key worked examples. Students are encouraged to participate in shared solutions of problems, to ask questions in search of deeper understanding and to reflect on where new learning may develop later. Students are supplied with exercise books to use for practice exercises and notes. Textbooks published by Pearsons (Edexcel) are often used for practice both in and out of lessons. The department has a comprehensive VLE set up through Moodle, which has a rich collection of supporting resources as well as topic based Review homeworks based on exam board questions and examiners’ mark schemes.

**Working expectations:**

You are expected to spend time after every lesson reflecting on the work covered and completing further practice (there is not sufficient time in lessons to put in the necessary practice). Only by working through questions do you establish mastery of techniques and by developing strategies to solve problems, secure learning take place. However independent you may be as a learner, opportunities to discuss your reasoning with other students and compare strategies will all lead to refined practice and deeper understanding. Both A level Maths and Further Maths place a far greater emphasis on proof than GCSE and so the quality of your mathematical writing is crucial – students are therefore expected to make detailed comparisons between their own solutions and the model solutions that are shared with them.

**What 100% effort in these subject looks like:**

* You need a level of stamina that means you will put in the routine practice needed, usually involving about the same time out of lessons as that committed during lesson time.
* A curiosity about the subject which means you always want to know why, as well as how.
* A self-discipline to look for any mistakes in your work and to uncover their cause and resolve them.
* Sufficient resilience that means you might walk away from a challenging problem to make a cup of tea, but only to come back to it again later and seek out further support if you still haven’t found a method of solution.
* An appreciation of what rigour in mathematical reasoning looks like, so that your own mathematical writing continues to develop and improve.
* Seeking out further exam practice through a variety of online options.
* An interest and appreciation of how the mathematics you are learning relates to the other subjects you are studying, as well as future university and career aspirations. This includes asking questions to ascertain some differences in notation and language between subjects and applications.

**Book and Folder Policy:**

The maths department supply students with A4 exercise books, which are used in lessons and at home for class notes and textbook exercises.

*Each exercise book should:*

* reflect that regular practice is taking place outside lessons, building on from work done in class.
* be well organised with lesson and content headings as well as highlighted examples and notes that can be easily revisited when work is reviewed.
* evidence regular reflective marking of all work such that mistakes are quickly identified and resolved

and useful learning is gained from these errors (students are asked to mark in a different coloured pen to help emphasise errors for future reference).

All students must also have an assessment folder, which is shown to staff when requested:

*Your folder should have:*

* + Dividers separating the work into sections, organised by topic and clearly distinguishing Maths from Further Maths content.
	+ Moodle chapter review homework tasks, marked by the student, submitted for teacher checking and sorted by topic.
	+ All chapter tests, organised with the corresponding homework, which have been teacher marked. Corrections and any other necessary development tasks having been completed.
	+ Mock exams which have been teacher marked and related corrections and model solutions are included.
	+ Evidence of folder review sheets where peer assessment informs ways to improve your folder.

**What Marking looks like:**

* Student self-marking of routine practice exercise is visible confirming correct solutions but also builds on errors by repeating the aspect of the question that has caused the mistake (self-reminder notes can also be helpful).
* Teacher checking of chapter reviews will be recorded classified by whether or not the task shows evidence of a student’s confident understanding and application of the content learned and appropriate reflection from marking, making appropriate use of the given markschemes.
* Teacher marking will follow examiners’ practice, distinguishing between marks awarded for correct methods (M), from accuracy marks (A) or those independent of methods (B). This reflects the importance placed on proof in A Level Maths and Further Maths, such that evidence of reasoning and mathematical justification, using good mathematical notation is required.

**What Homework looks like:**

* Routine continued practice, usually set from textbook exercises. Further practice exercises are also supplied in Moodle for each topic.
* Further exploration of reasoning by looking at other sources such as online tuition videos and looking at solution bank model solutions. This can also be valuable time spent talking through your thinking with another student.
* Solving and reviewing topic based past exam questions in end of chapter Moodle homework tasks.
* Practice exam papers, helping to develop confidence in managing identification of the maths used to solve a wide variety of problems

**Specification at a glance:**

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| **Y1 Pure Maths Content Summary** | **Y1 Applied Maths Content** |
| **Algebra and functions (part 1)** | **Statistics** |
| Algebraic expressions – basic algebraic manipulation, indices & surdsQuadratic functions – factorising, solving, graphs and the discriminantsEquations – quadratic/linear simultaneous | **Data presentation and interpretation (part 1):** Calculation and interpretation of measures of location; Calculation and interpretation of measures of variation; Understand and use coding |
| **Further algebra**The binomial expansionAlgebraic division, factor theorem and proof | **Statistical distributions:** Use discrete distributions to model real-world situations; Identify the discrete uniform distribution; Calculate probabilities using the binomial distribution (calculator use expected) |
| **Differentiation**Definition, differentiating polynomials, second derivativesGradients, tangents, normals, maxima and minima | **Probability:** Mutually exclusive events; Independent eventsIntroduction to sampling terminology; Advantages and disadvantages of samplingUnderstand and use sampling techniques; Compare sampling techniques in contextLanguage of hypothesis testing; Significance levels Carry out hypothesis tests involving the binomial distribution |
| **Integration**Definition as opposite of differentiation, indefinite integrals of *xn*Definite integrals and areas under curves | **Data presentation and interpretation (part 2):** Interpret diagrams for single-variable data; Interpret scatter diagrams and regression lines; Recognise and interpret outliers; Draw simple conclusions from statistical problems |
|  | **Mechanics** |
| **Vectors (2D)**Definitions, magnitude/direction, addition and scalar multiplicationPosition vectors, distance between two points, geometric problems | Introduction to mathematical modelling and standard S.I. units of length, time and massDefinitions of force, velocity, speed, acceleration and weight and displacement; Vector and scalar quantities  |
| **Coordinate geometry in the (*x*, *y*) plane**Straight-line graphs, parallel/perpendicular, length and area problemsCircles – equation of a circle, geometric problems on a grid | Graphical representation of velocity, acceleration and displacement  |
| **Trigonometry**Trigonometric ratios and graphsTrigonometric identities and equations | Motion in a straight line under constant acceleration; *suvat* formulae for constant acceleration; Vertical motion under gravity Newton’s first law, force diagrams, equilibrium, introduction to **i**, **j** system Newton’s second law, ‘*F* = *ma*’, connected particles (no resolving forces or use of *F* = *μR*); Newton’s third law: equilibrium, problems involving smooth pulleys  |
| **Algebra and functions (part 2)**Inequalities – linear and quadratic (including graphical solutions)Graphs – cubic, quartic and reciprocalTransformations – transforming graphs – f(*x*) notation | Variable force; Calculus to determine rates of change for kinematicsUse of integration for kinematics problems i.e.   |
| **Exponentials and logarithms:**  Exponential functions & natural logs |  |

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| **Y1 Further Core Pure Maths Content Summary** | **Y1 Optional Applied Further Maths Content** |
| **Complex numbers** | **Further Statistics** |
| Introduction of complex numbers, basic manipulationArgand diagramsModulus and argumentLoci Complex conjugate, division & solving polynomial equations | **Discrete probability distributions**Mean and variance of discrete probability distributions &extension of expected value function to include E(g(X)) |
| **Matrices**Matrix addition, subtraction and multiplicationInverse of 2×2 and 3×3 matricesSimultaneous equationsLinear transformations | **Poisson and binomial distributions**The Poisson distributionMean and variance of the binomial and Poisson distributionsPoisson distribution as an approximation to the binomial distributionHypothesis test for the mean of a Poisson distribution |
| **Series**Sums of series | **Chi squared tests**Chi squared tests for goodness of fit |
| **Algebra and functions**  | **Further Mechanics** |
| Roots of polynomial equationsFormation of polynomial equations | **Momentum and impulse**Momentum and impulse; derivation of units and formulae Impulse-momentum principle. Conservation of momentum applied to collisions and ‘jerking’ string problems |
| **Proof**Proof by mathematical induction | **Elastic collisions in one dimension**Direct impact of elastic spheres. Newton’s law of restitution. Loss of kinetic energy due to impactProblem solving (including ‘successive’ impacts) |
| **Vectors**Vector and Cartesian equations of a line and a planeScalar productProblems involving points, lines and planes | **Work, energy and power**Work, kinetic energy; derivation of units and formulaePotential energy, work-energy principle, conservation of mechanical energy, problem solvingPower; derivation of units and formula |
| **Calculus** | **Decision Mathematics** |
| Volumes of revolution | **Algorithms and graph theory**Introduction to algorithms and graph theorySorting algorithmsMinimum connectors (spanning trees)Dijkstra’s algorithmRoute inspection problemPlanarity algorithmFloyd’s algorithm  |
|  | **Linear programming**Formulation of problemsGraphical solutions |
|  | **Critical path analysis**Activity networks; precedence tablesCritical path algorithm; earliest and latest event timesTotal float; Gantt charts |

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| **Y2 Pure Maths Content Summary** | **Y2 Applied Maths Content** |
| **Algebraic and partial fractions**  | **Statistics** |
| Simplifying algebraic fractionsPartial fractions | **Probability**Using set notation for probability  |
| **Trigonometry** **(part 1)**Radians (definition and exact values)Small anglesSecant, cosecant and cotangent (definitions, identities and graphs)Inverse trigonometrical functionsCompound angle formulae  | Conditional probabilityQuestioning assumptions in probability |
| **Differentiation**Differentiating sin *x* and cos *x* from first principlesDifferentiating exponentials and logarithmsDifferentiating products, quotients and implicit functions.Second derivatives (rates of change of gradient, inflections)Rates of change problems (including growth and kinematics) | **The Normal distribution**Understand and use the Normal distributionUse the Normal distribution as an approximation to the binomial distribution Selecting the appropriate distribution Statistical hypothesis testing for the mean of the Normal distribution |
| **Integration**Integrating *xn*, exponentials and trigonometric functionsIntegration by substitutionIntegration by partsUse of partial fractionsAreas under graphs or between two curvesThe trapezium ruleDifferential equations  | **Regression and correlation**Change of variableProduct Moment Correlation Coefficients Statistical hypothesis testing for zero correlation |
| **Proof:** including proof by deduction and proof by contradiction | **Mechanics** |
| **Functions and modelling**Modulus functionComposite and inverse functionsTransformationsModelling with functions | **Further kinematics (part 1)**Constant acceleration (equations of motion in 2D; the **i**, **j** system)**Applications of kinematics**  Projectiles |
| **Series and sequences**Arithmetic and geometric progressions Sigma notationRecurrence and iterations | **Forces at any angle:**  Resolving forcesFriction forces (including coefficient of friction *µ*) |
| **The binomial theorem** Expanding (*a* + *bx*)*n* for rational *n*;Expansion of functions by first using partial fractions | **Moments**Forces’ turning effect |
| **Trigonometry (part 2)**Arcs and sectorsCompound and double (and half) angle formulaeHarmonic form*: R* cos (*x* ± *α*) or *R* sin (*x* ± *α*)Proving trigonometric identities Solving problems in context (e.g. mechanics)  | **Applications of forces**Equilibrium and statics of a particleDynamics of a particleEquilibrium and statics of a particle (including ladder problems) |
| **Parametric equations**Definition and converting between parametric and Cartesian formsCurve sketching and modellingDifferentiating parametric functionsIntegrating functions/ Areas under curves defined parametrically | **Further kinematics (part 2)**  Variable acceleration (use of calculus and finding vectors $\dot{r}$ and $\ddot{r}$ at a given time) |
| **Numerical methods**Location of rootsSolving by iterative methods ( ‘staircase and cobweb’ diagrams)Newton-Raphson method |  |
| **Vectors** (**3D):**  Use of vectors in three dimensions; knowledge of column vectors and **i**, **j** and **k** unit vectors |  |

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| **Y2 Core Pure Further Maths Content Summary** | **Y2 Optional Applied Further Maths Content** |
| **Complex numbers** | **Further Statistics** |
| Exponential form of complex numbersDe Moivre’s theoremThe *n*th roots of *z* = *r*ei*θ* and complex roots of unity | **Geometric and negative binomial distributions**The geometric distribution, its mean and varianceThe negative binomial distribution, its mean and varianceHypothesis test for the parameter *p* of a geometric distributionChi squared tests for goodness of fit of a geometric distribution |
| **Polar coordinates**Convert between Cartesian and polar and sketch *r*(*θ*)Area enclosed by a polar curve | **The Central Limit Theorem**Applications of the Central Limit Theorem |
| **Hyperbolic functions**sinh *x*, cosh *x*, tanh *x* and their inversesLogarithmic forms of the inverse hyperbolic functions and integrate functions of the form $(x^{2}+ a^{2})^{- \frac{1}{2}}$ and$(x^{2}- a^{2})^{- \frac{1}{2}}$ | **Probability generating functions**Definitions, derivations, applications and use to find the mean and varianceUse of the probability generating function for the negative binomial, geometric, binomial and Poisson distributionsProbability generating function of the sum of independent random variables |
| **Further algebra and functions (series)**Method of differencesMaclaurin series | **Quality of tests**Type I and Type II errorsThe size and power of a testThe power function |
| **Further calculus** | **Further Mechanics** |
| Improper integralsMean value of a functionIntegrate using partial fractionsDifferentiate inverse trigonometric functions and integrate using trigonometric substitutionsFurther volumes of revolutions | **Elastic strings and springs and elastic energy**Hooke’s law and definition of modulus of elasticity. Derivation of elastic potential energy formula.Problem solving: equilibrium and using the work-energy principle |
| **Differential equations**Integrating factors to solve first order differential equationsSecond order differential equations of the form *y*′′ + *ay′ + by* = f(*x*)Modelling | **Momentum and impulse**Momentum as a vector (**i**, **j** problems)Impulse-momentum principle in vector form |
|  | **Elastic collisions in two dimensions**Oblique impact of a smooth sphere with a fixed surface Successive oblique impacts of a sphere with smooth plane surfacesOblique impact of two smooth spheres of equal radius |
|  | **Decision Mathematics** |
|  | **Algorithms on graphs**Travelling salesman problem |
|  | **Linear programming**Formulation of problemsSimplex algorithmBig-M and two-stage Simplex |
|  | **Critical path analysis (part 2)** Resource histogramsScheduling |

**Examinations – at the end of the two year course**

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| **A level Mathematics** |
| **Paper 1:**Pure Mathematics 33%, 2 hours, 100 marks | Any pure content can be assessed on either paper |
| **Paper 2:** Pure Mathematics 33%, 2 hours, 100 marks |
| **Paper 3:** Statistics and Mechanics33%, 2 hours, 100 marks | Section A: Statistics (50 marks)Section B: Mechanics (50 marks) |

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| **A level Further Mathematics** |
| **Paper 1:**  Core Pure Mathematics 125%, 1 hour 30 seconds, 75 marks | Any A level core pure mathematics content assessed |
| **Paper 2:**  Core Pure Mathematics 225%, 1 hour 30 seconds, 75 marks |
| **Paper 3:**  Further Mathematics Option 125%, 1 hour 30 seconds, 75 marks | Students take two optional papers with options available in:* Further Statistics
* Further Mechanics
* Decision Mathematics
 |
| **Paper 4:**  Further Mathematics Option 225%, 1 hour 30 seconds, 75 marks |

**Summer preparation**

The Step up to A level Maths from Higher GCSE is very manageable if you had mastered all of the content before your GCSE but you need to be ready for a fast paced, challenging start.

The list below summarised the skills we expect you to be confident with:

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| **Number** | **Algebra Basics** | **Geometry** |
| Working with fractions | Expanding brackets | Pythagoras’ theorem |
| Laws of indices | Terms in expressions and equations | Trigonometry basics |
| Surds | Changing the subject of a formula | The Sine amd Cosine Rules |
|  | Solving Linear equations | Vectors |
| **Algebraic Graphs** | Solving inequalities |  |
| Equation of a straight line | Solving simultaneous equations | **Statistics & probability** |
| Parallel & perpendicular lines | Factorising | Histograms & cumulative frequency |
| Non-linear graphs | Solving quadratic equations | Sampling and averages |
| Transforming graphs | Completing the square | Probability & tree diagrams |
| Trigonometry graphs |  |  |
| Graphing inequalities |  |  |

The step up to A level Further Maths will vary according to students’ prior experience. Those students who have taken GCSE Statistics or the level 2 Further Mathematics qualification will already be familiar with aspects of the A level content such as: ***binomial expansion, complex numbers, differentiation, matrices ; standard deviation; standardisation of data, Product moment Correlation Coefficient, binomial and normal distribution***

Summer preparation tasks

The purpose of giving you a summer bridging task is:

1. To provide a bridge from level 2 to level 3 study, and lead into the early stages of the course.
2. To engage you in independent learning which is required at level 3.
3. To encourage you to develop your work ethic and commitment to study.
4. To measure your suitability for the course and assess your initial levels of achievement.

**Task**: Review your GCSE skills from the list above, identifying any that may benefit from review and practice, selecting the appropriate practice worksheets from the ***Mathematics bridging task zip file***. Then use the link: <https://nrich.maths.org/1403> to work through an ***Introduction to Complex numbers***. If you are confident with all of the GCSE topics above but do not have any experience of the level 2 further maths and statistics skills then select some of the other new topics you are about to meet at A level from the ***Further Maths bridging task*** folder.

  

You will need a ***Casio Classwiz calculator*** for A level model fx991EX

These retail at between £25 and £35

Note that Casio do produce more advanced models that also include graphing software.

The CGP ***Head Start to A-Level Maths*** book offers a thorough review of the topics from GCSE that you will be using at A level

**Potentially useful websites:**

Studying Maths beyond GCSE course choices:

<https://amsp.org.uk/students/gcse/what-next>

The Jump - GCSE to A Level Bridging Course

<https://www.youtube.com/watch?v=mY7tn3NT9MQ>

### GCSE to A-Level Maths: Bridging the Gap NEW - [TLMaths](https://www.youtube.com/channel/UCCgGyPD6MYQcHuMIc-Kv-Uw)

<https://www.youtube.com/watch?v=QWaatt3t76k&list=PLg2tfDG3Ww4vuevQXAC13EyUvpVa0UrNa>

**Link to the Edexcel Mathematics Specification:**

<https://qualifications.pearson.com/en/qualifications/edexcel-a-levels/mathematics-2017.html>

**Link to the Edexcel Further Mathematics Specification:**

<https://qualifications.pearson.com/en/qualifications/edexcel-a-levels/mathematics-2017.html#%2Ftab-AlevelFurtherMathematics>